
Ahmed KSAIER*, Isabelle CRISTIANI-D’ORNANO**

Abstract: We observe from the late 1990s an increasing phenomenon of volatility on these following markets: Oil (WTI price), Foreign Exchange (nominal Euro/Dollar), Stock Market (S&P 500 Index) and Bond market (U.S.10-Year). After seizing the concept of volatility and overcoming its first definition of risk measure, we have evaluated their interdependencies from a VAR model, we have investigated the presence of long memory phenomenon in these series and we have carried out their forecasted trajectories from FIGARCH model. This paper is presented as follows: Section 1 opens on a definition of the volatility, Section 2 examines the interdependence of the studied markets; Section 3 provides a FIGARCH model in order to capture the dynamics and predict future market volatilities changes and Section 4 concludes.

Keywords: Volatility, Long Memory, FIGARCH, Forecasting.

JEL Codes: C22, C53, G17

INTRODUCTION

The 2008-2009 financial crisis raised by its size and its economic and social consequences many reactions, questions and concerns in public opinion about both organization and soundness of financial systems. The global financial crisis has seen the biggest drop in global economic activity in the modern era. In 2009, most major developed economies were in a deep recession. Predicting the financial crisis has not been possible by using early warnings models that some economists had developed from several stress indicators. Although these early warning systems have provided important

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and useful information in terms of detection of degree of vulnerability to crises and in terms of crisis occurrence, they were less effective under the 2008 crisis.

Given the shortcomings of early warning indicators models and existing international turmoil, it seems appropriate to understand the concept of volatility on financial markets in order to capture price movements.

Intuitively, the term volatility measures the uncertainty about the future and refers to the concept of risk. According to Galai (1991), volatility measures the standard deviation of the financial asset distribution and is not equivalent to the risk of losing money. It is rather from the deviation that the risk to be below a certain threshold can be measured. Generally speaking high volatility creates in the mind of many people the strong idea that the market functions abnormally, which requires a correction. Moreover, for those which are daily concerned with the phenomenon of volatility, volatility is the crucial indicator on which each investment decision, resource allocation and portfolio construction is built.

Do the increasing trend of volatility and the multiplication of the peaks observed result from cyclic phenomena; belong to the characteristics of the current period (stock market decline, challenging the criteria for assessing the value of assets, high debt companies)? Or do these trends are due to more structural factors, i.e. related to financial markets or management techniques implemented by investors?

The financial innovation and the increasing sophistication of the techniques and instruments at the disposal of actors are regularly suspected of being responsible for the formation of the volatility on financial markets. This suspicion aims, in particular, the assets of optional nature. So, one of the common characteristics to major financial instruments developed during last years is the fact that they integrate, explicitly or implicitly, an optional component (convertible bonds, assets offering a guarantee in capital or a guarantee of performance, contingent bond clauses)

In a general way, volatility refers to the price trend change in an unforeseen period, in response to new information or to an external shock being able to affect the evolution of the fundamental market factors. Volatility measures the degree of increase or reduction in the prices during one short period. It is not defined compared to the price level, but by its degree of variation. High levels of the prices do not mean that the prices are very volatile like the low levels of price can also reveal a strong volatility. In other words, the level of volatility is not influenced by the direction of the prices trend.

On the financial front, one distinguishes two types of volatility: historical volatility and implicit volatility. Historical volatility is founded on the past behaviour, which is to say on the last variations of the financial asset price. It gives the level of volatility reached in the past on the basis of the last trend of the underlying asset price. Implicit
volatility results from market anticipations on the future price variations or from the premium of the underlying asset.

Measurements of volatility are thus regarded as a barometer of the investors’ mood. High level of volatility usually indicates a great nervousness of the market whereas a low level corresponds to a weak risk of the market, therefore a taking risk tendency. It is consequently the estimate of this component of volatility that caused various mathematical and econometric methods.

A persistent and high level of volatility is one of the statistical characteristic of the volatility.

In other words, the important variations of the assets price do not suddenly stop after the consideration of important new information, but tend to persist. This dynamics means that the perception of high volatility influences volatility anticipations by actors on markets. Observable phenomenon at the time of the financial crises, within similar markets or between various markets, the transmission of volatility is both a sign and a factor of tensions.

Research in financial market volatility has been concentrating on modelling and less on forecasting. Working on combined forecast is rare, probably because the groups of researchers working on time series models and option pricing do not seem to mix. What has not yet been done in the literature is to separate the forecasting period into ‘normal’ and ‘exceptional’ periods. It is conceivable that different forecasting methods are better suited to different trading environment and economic conditions.

Recent literature has shown empirical evidence of an increasing integration degree among stock markets, probably facilitated by rapid transmission of technology. Understanding and measuring these interdependencies is important for portfolio selection, hedging, and accurate assessment of risk in general. In particular, crisis seems to increase the frequency and magnitude of co-movements (joint high gains or joint extreme losses) among stock indexes, risky assets, and economic indicators.

Financial market volatility is known to cluster. A volatile period tends to persist for some time before the market returns to normality. The ARCH (Autoregressive Conditional Heteroscedasticity) model proposed by Engle (1982) was designed to capture volatility persistence in inflation.

Volatility is a key input in many financial applications, including optimal portfolio construction and risk management. In addition, volatility itself is also a tradable asset that has attracted numerous investors. Certainly, correctly modelling volatility has become a crucial task in finance. However, volatility per se is not directly observable and it is necessary to employ a reasonable proxy to empirically assess the links. Ever since Engle (1982) introduced the ARCH model to explicitly parameterize the volatility process (including an extension to the GARCH model developed by Bollerslev, 1986), there have
been many empirical investigations into the temporal link between returns and volatility that employ ARCH-type models. In particular, Engle et al. (1987) develop the ARCH-M model to include volatility directly in the return generating process, and thus allow testing the relation between returns and volatility.

Taylor (1987) study was one of the earliest ones to test the predictive power of GARCH. Although this earlier investigation of Taylor, Akigray (1989) is more commonly cited in many subsequent GARCH studies. In the following decade, there were no fewer than 20 papers that test GARCH predictive power against other time series methods and against option using volatility forecasts. The majority of these forecast volatility dealt about stock indices and exchange rates.

Volatility persistence is a feature that many time series models are designed to capture. A GARCH model features an exponential decay in the autocorrelation of conditional variances. However, it has been noted that squared and absolute returns of financial assets typically have serial correlations that are slow to decay, similar to those of an I(d) process. A shock in the volatility series seems to have very ‘long memory’ and impacts on future volatility over a long horizon. The integrated GARCH (IGARCH) model of Engle and Bollerslev (1986) captures this effect, but a shock in this model impacts upon future volatility over an infinite horizon and the unconditional variance does not exist for this model.


In this study we emphasize the leading volatility in the problem of financial crises forecasting, which implies the necessity for modelling volatility in order to understand its dynamics and try to predict its future trajectories. This allows avoiding pressure and stress on the financial market going to lead to a crisis.

The remainder of the paper is organized as follows. Section II examines the interdependence of the studied markets; Section III provides a model FIGARCH in order to capture the dynamics and predict future market volatilities changes and Section IV concludes.
INTERDEPENDENCE

Data

These data relate to the financial market (S&P 500 Index), the oil market (price of the WTI), the bond market (10-Year Treasury constant maturity rate) and the foreign exchange (nominal Euro/Dollar) from January 1st, 1999 to November 6th, 2009. For these last data, our daily data resulting from Federal Reserve.

The oil price rise has been fed by the strong world economic growth (in particular Chinese one) during the years 2004–2007, the strongest growth for four decades. Its evolution is however closely related to the dollar and a negative correlation is observed between these two variables (Bénassy-Quéré, Mignon and Penot 2005). The investors taking note of this correlation reinforced it by their actions on the market by using oil as cover against the fall of the dollar. Strong monetary creation also fed the rise in the crude oil prices, and then the latter very strongly fell since August 2008, thus accompanying the sharp appreciation of the dollar.

- Historical observations on the four markets:
The SP500 Index became very volatile since the birth of the Internet bubble until it’s bursting on August 2000, which caused a fall of the prices of the stock market until the beginning of the year 2003. These prices regained and even exceeded the same peaks in 2007 before starting a new decline in July 2007 caused by the crisis of the subprimes in the United States. These banking and financial crises were transmitted to the real sphere in the second half year of 2008, which saw the slump of many great banking institutions and the fear of the systemic risk occurrence. In view of these risk and increased fall of the stock market, the Central banks - which had already largely intervened- have adopted unconventional monetarist policy like quantitative easing. The ECB, on its own, supports States, which do not belong to its area. These insurances offered by the Central banks as well as the abundant liquidity allowed the increase of stock market starting from the beginning of March 2009.
The evolution of the yield of the 10-Year Treasury bill is both naturally function of the Fed Fund rate and inflationary expectation of agents. After reaching nearly 7% at the beginning of the year 2000, following the inflationary pressures in the United States, this interest rate started to decrease following expectations of economic deceleration. This latter was caused by the inversion of yield curve, which announced the slump of 2001. The recession involved the decline of this interest rate and conversely its rise accompanying the strong increase of inflation since 2004. Strong global monetary creation limited interest rate rise. However, during this same period, the Federal Reserve applied its restrictive monetary policy, and then induced a new inversion of the yield curve, announcing by this way the recession of 2008. It’s very strong width, the deflationary risk at the end of 2008 and especially quantitative easing led the 10-Year interest rate at its historical low levels.
The Euro launched in 1999 whereas the dollar knew one period of strong appreciation since the mid-90’s. This appreciation was mainly explained by a favourable differential of productivity in the United States. Nevertheless, the acceleration of the American current account deficit growth beyond the threshold of 3% of the GDP led the American authorities to take measures to obtain from markets a depreciation, which became effective from March 2002. This depreciation is fed until 2007 by the strong increasing U.S. current account deficit, punctuated by some phases of appreciation, in particular during 2005, at the time of an increasing differential of interest rate in favour of the United States. The strong appreciation of the dollar during the second half of 2008 is consecutive with the increasing concern of the crisis contagion to the worldwide economy, which involved a phenomenon of flight to safety in favour of the American dollar. On December 2009, we observed both Dubai default fear and the downgrading of the Greek debt.

**Statistical characteristics**

- **Descriptive statistics**
All data (oil, S&P 500, 10-Year U.S. interest rate and the Euro/Dollar) are converted into nominal daily series of return (in %):
where 
\[ r_t = 100 \ln \left( \frac{S_t}{S_{t-1}} \right) \] for \( t = 1, 2, \ldots, T \),

in which 
\[ r_t \] is the return at the time \( t \), 
\( S_t \) the current price in \( t \) and 
\( S_{t-1} \) is the previous price.

The daily volatility (variance) is measured by the daily squared return \( r_t^2 \).

<table>
<thead>
<tr>
<th>Series</th>
<th>S&amp;P 500</th>
<th>OIL WTI</th>
<th>EURO DOLLAR</th>
<th>10-Y US Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1201.312</td>
<td>47.80692</td>
<td>1.172816</td>
<td>4.581535</td>
</tr>
<tr>
<td>Skewness</td>
<td>-.310026</td>
<td>1.199994</td>
<td>.0318605</td>
<td>.1513097</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.234317</td>
<td>4.261557</td>
<td>2.032923</td>
<td>2.999809</td>
</tr>
</tbody>
</table>

Table 30 Descriptive statistics

It arises from the analysis that except the volatility of the oil price, other volatilities are “normally” distributed.

- **Test of stationarity**

  All the return series are subjected to the two unit tests, ADF (Augmented Dickey Fuller) and Philips-Perron one in order to determine if the stationarity, the integration and the fractional parameter of integration can be considered in these daily data.

  Within the framework of these ADF and PP tests, the null assumption (HO) stipulates that the time series contains a unit root, that is to say I (1). The empirical results presented in the Table 31 show in all cases the existence of high negative values, which translates a rejection of the null assumption for a level of significance of 5%. These series are thus significantly stationary.

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Intercept and trend</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-25.066</td>
<td>-25.054</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
<td>(-3.41)</td>
</tr>
<tr>
<td>Oil Price</td>
<td>-24.852</td>
<td>-24.839</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
<td>(-3.41)</td>
</tr>
<tr>
<td>Euro/Dollar</td>
<td>-22.561</td>
<td>-22.571</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
<td>(-3.41)</td>
</tr>
<tr>
<td>10-Year US Interest Rate</td>
<td>-27.24</td>
<td>-27.21</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
<td>(-3.41)</td>
</tr>
</tbody>
</table>

Table 31 Stationarity Tests
- **ARCH Test**
  
  ARCH (Autoregression Conditional Heteroscedasticity) models make it possible to modelize chronicles (financial among others), which have the characteristic to have an instantaneous volatility (variance) belongs to the past. From where the possibility of carrying out a modelling of the chronicle in term of mean and variance; this test rests either on a test of Fisher or on a test of Lagrangian multiplier (LM).

  - **Test of Lagrange Multiplier in order to determine ARCH effects**

    ARCH Model has the form of an autoregression one. Engle (1982) proposes the test of the Multiplier of Lagrange (LM) in order to test the existence or not of an ARCH behaviour within the regression. The statistical test is given by TR², where R² is the coefficient of determination and T is the sample size. The null assumption (HO) stipulates that there are no ARCH effects and its distribution asymptotically follows a Chi-square distribution with p degrees of freedom.

    If LM > χ² (p) with p degrees of freedom to a threshold α = 0.05, then HO is rejected, from where the justification of an ARCH (p) model.

    

    **Table 32 Lagrange Multiplier Test**

    |         | RSP | ROIL | REUDO | R10Y-T |
    |---------|-----|------|-------|--------|
    | LM      | 32.246 | 2.036 | 2.277 | 6.748  |
    | Significance level | **0.00000629** | **0.00054779** | **0.04832632** | **0.00019326** |

    The results pos a level of significativity Q = 0.0000, from where the indication of the existence of ARCH effects in all series.

- **Detection of the long memory by the exponent of Hurst**

  Fractal geometry helps us to see the economic world from a different point of view. This mathematical approach considers that a shape is composed of a basic pattern which is multiplied at infinite scale. The fractal dimension is directly related to the Hurst exponent: a small Hurst exponent has a higher fractal dimension (a rougher surface) whereas a larger Hurst exponent has a smaller fractional dimension (a smoother surface). These Brownian walks can be generated from a defined Hurst exponent. Our interest in the Hurst exponent is motivated by its power for estimating forecasting in financial time series.

  In its “Rescaled Range Analysis” (R/S Analysis) Hurst discusses the analysis of extended standard range where R series are centered and integrated: R = (max X (t, τ) - min X (t, τ)) with 1 ≤ t ≤ τ. R is divided by its standard deviation S (t, τ). Hurst shows that this normalized range R / S is relative to the interval considered by the relationship: R / S = (aτ)H where a is a constant, τ expresses some multi-year period and H the Hurst exponent ranging from 0 to 1. So, 0< H <1. Hurst exponent can justify the existence of
long memory within the time series. In the event of presence of long memory, these series are fractionally integrated. The presence of long memory is attested if $0.5 < H < 1$ i.e. the random walk will be a long memory process.

When $H = 0.5$, i.e. $\alpha = (R / S)^2$, we are in the presence of white noise, a random walk.

When $H > 0.5$, we are witnessing the phenomenon of persistence: the "noise" is not random, and each observation is its memory footprint. This is a fractional Brownian motion. More $H$ approaches 1, more the strength of memory acts. The correlation of long-term reflects the fact that the variation of the series tends to follow the same trend. This means in our study that if a price increases, the probability that it is still growing is strong.

When $H < 0.5$, we are witnessing a phenomenon of anti-persistence. Here, the correlation between long-term observations is negative, reflecting an alternation of positive and negative variations (mean reversion).

According to our results, Table 33 presents the test of Hurst for the volatility of each market. It appears a phenomenon of high persistence, which translates the existence of an effect of long memory on each market.

### Table 33 Hurst Test

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500 Index</th>
<th>Oil price WTI</th>
<th>Euro/Dollar Exchange Rate</th>
<th>10-Y Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return</strong></td>
<td>0.60</td>
<td>0.53</td>
<td>0.65</td>
<td>0.57</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>0.89</td>
<td>0.72</td>
<td>0.75</td>
<td>0.92</td>
</tr>
</tbody>
</table>

- **Volatility on the four markets**

The volatility of the oil price is, on the end of the studied period, very related to the evolution of the dollar which knew a peak of appreciation at the beginning of August 2008 until the end of October 2008. The reason is due to a flight to safety related to the worry concerning the contagion of the American financial crisis, then economic one, on the worldwide economy. The behaviour of the market operators using oil as cover on the dollar reinforced the opposite correlation between these two variables.
On the end of the studied period, the S&P 500 volatility increased at the beginning of the crisis of the subprimes in July 2007 and knew its peaks at the time of the stock market immediately crashed which the followed fall of Lehman Brothers (September 2008). It slowed down after the warning of the G7 on October 2008 to guarantee in particular the interbank loans but is remained high until the central banks - and in particular the ECB - do not possibly provide support to the emergent countries in Europe in the event of a failure. Then, principal fear was the occurrence of a defect into an emerging economy.

The 10-Year US Interest Rate volatility became extremely strong at the time of the Lehman Brothers bankruptcy in September 2008 with a decrease of the return due to a strong concern on the economic situation and due to the fear of deflationary risk in this end of 2008. The bond market sharply moved on December 16th, 2008 when the Fed announced its intention to apply a quantitative easing policy. The peak of volatility was reached on March 18th, 2009 when the Federal Reserve announced that it decided, rather than to anticipate by the markets, to buy for $300 billion of longer-term Treasury securities over the next six months to improve conditions on private credit markets.
The Euro/Dollar volatility, over the last period, is much related to that of Treasury Bond. Indeed, the principal peaks occurred on December 16th 2008 when the Fed announced that it was going to follow an unconventional monetary policy of quantitative easing and on March 18th 2009 when the Fed surprised the markets by the timing and the width of this quantitative policy.

**VAR Estimate**

We use the VAR approach in order to evaluate interdependencies and impacts between these four markets. The results of estimations are reported in this following table.

**Table 34 Results**

<table>
<thead>
<tr>
<th></th>
<th>Oil price WTI</th>
<th>S&amp;P 500 Index</th>
<th>10-Y US Interest Rate</th>
<th>Euro/Dollar Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil price WTI</td>
<td>Lag 1: 0.099*</td>
<td>Lag 2: 0.051*</td>
<td>0.012*</td>
<td>0.026*</td>
</tr>
<tr>
<td></td>
<td>Lag 1: 0.213*</td>
<td>Lag 2: 0.171*</td>
<td>0.087*</td>
<td>0.019*</td>
</tr>
<tr>
<td></td>
<td>Lag 1: 0.231*</td>
<td>Lag 2: 0.109*</td>
<td>0.126*</td>
<td>0.083*</td>
</tr>
<tr>
<td></td>
<td>Lag 1: 0.791*</td>
<td>Lag 2: 2.230*</td>
<td>0.203*</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>Lag 1: 0.203*</td>
<td>Lag 2: 0.293*</td>
<td>0.114</td>
<td>0.094*</td>
</tr>
</tbody>
</table>

It appears from the VAR study that:
• The volatility of the oil price (WTI) is influenced by itself and by those of the other markets (S&P 500), interest rate (10-Years) and exchange rate (Euro/Dollar).

• The volatility of financial market (S&P 500) is nourished by all other volatilities except by those related to oil market and those of the 10-Year US interest rate on the second lag.

• The volatility of the 10-Year US interest rate is fed by itself, by that of the oil price and by that of the financial market on the second lag. Conversely, the volatility of the Euro/Dollar price does not have any influence on it.

• The volatility of the Euro/Dollar price is caused by itself, by 10-Year interest rate and on the first lag by the financial market. For as much, the volatility of this last does not act on the second lag and the volatility of the oil price never acts.

- Study of causality

According to the Granger causality tests, we obtained the following results:

• The volatility of the oil market WTI is caused overall by the three markets which are financial market (S&P 500), interest rate market (the 10-Year US) and the foreign exchange (Euro/Dollar).

• The volatility of the financial market is influenced by the three other markets but is not directly by the oil market. It appears that there are not any causal relations between volatilities of the financial and oil markets.

• The volatility of the 10-Year interest rate is influenced by those of the three other markets but unilaterally is not by the volatility of the Euro/Dollar. In addition this volatility influences the three other volatilities.

• As for the volatility of Euro/Dollar, it seems that causal connections are overall established and in particular starting from the interest rate market but there is no unilateral source starting from the oil and financial markets.

It arises from our results that the oil, financial, interest rate and the exchange rate markets are all interdependent.

**Modelling**

**Empirical modelling**

By considering the existence of the phenomenon of long memory, it is then necessary to measure the relevance of the FIGARCH model in order to modelize the volatility of the following series: S&P 500, WTI, 10-Y US. Interest Rate and Euro/Dollar.
• FIGARCH Modelling

FIGARCH (Fractionally Integrated Generalized Autoregression Conditional Heteroscedasticity) Model will enable us to modelize the volatility observed on the whole selected data because it has the property to post periods of high (low) volatilities which tend to be followed by similar sequences. In fact, FIGARCH model enables us to take into account the characteristics of the long memory.

Bailie and al. (1996) showed that FIGARCH (p, d, q) model posts an hyperbolic decrease in the process of volatility:

$$\Phi(L)(1-L)^d \varepsilon_i^2 = \omega + \left[1 - \beta(L)\right]v_i$$

Where $\omega > 0$, $0 < d < 1$, $\Phi(L) = [1 - \alpha(L) - \beta(L)](1-L)^d$ and all roots of $\Phi(L)$ and $[1 - \beta(L)]$ are located outside the unit circle. Alternatively, the expression of the conditional variance can be specified in the following way:

$$h_t = \omega \left(1 - \beta(L)\right)^{-1} + \left[1 - \left[1 - \beta(L)\right]^{-1} \Phi(L)(1-L)^d \varepsilon_i^2\right.$$  

$$= w + a(L)$$  

With $w = \omega \left(1 - \beta(L)\right)^{-1}$

$$a(L) = \sum_{i=1}^{\alpha} a_i L^i = 1 - \left[1 - \beta(L)\right]^{-1} \Phi(L)(1-L)^d$$

where $d$ is the fractional parameter of differentiation. ($0 < d < 1$)

After tests that we do not post here, it appeared that FIGARCH model offers a greater flexibility to model the conditional variance than models GARCH (when $d=0$) and IGARCH (when $d=1$) because the persistence of the shocks in the conditional variance or the degree of long memory is measured by the fractional parameter of differentiation $d$.

In this case: $0 < d < 1$, so FIGARCH offers a sufficient elasticity to take into account the intermediate degree of persistence to shocks.

**Results**

The estimated parameters by FIGARCH model are summarized in the below table:

<table>
<thead>
<tr>
<th>FIGARCH</th>
<th>WTI</th>
<th>SP500</th>
<th>Euro/Dollar</th>
<th>10-Year US. Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.09</td>
<td>0.0055</td>
<td>0.0014</td>
<td>0.0063</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>WTI</td>
<td>SP500</td>
<td>Euro/Dollar</td>
<td>10-Year US. Interest Rate</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>-------</td>
<td>-------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.443 (0.000)</td>
<td>0.332 (0.000)</td>
<td>0.371 (0.000)</td>
<td>0.457 (0.000)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.291 (0.000)</td>
<td>0.266 (0.000)</td>
<td>0.124 (0.000)</td>
<td>0.240 (0.000)</td>
</tr>
<tr>
<td>$d$</td>
<td>0.237 (0.005)</td>
<td>0.273 (0.001)</td>
<td>0.187 (0.002)</td>
<td>0.161 (0.002)</td>
</tr>
<tr>
<td>$D$</td>
<td>7.28 (0.000)</td>
<td>7.68 (0.000)</td>
<td>9.39 (0.000)</td>
<td>7.38 (0.000)</td>
</tr>
<tr>
<td>LB(20)</td>
<td>15.85 (0.066)</td>
<td>18.60 (0.048)</td>
<td>20.29 (0.037)</td>
<td>23.36 (0.022)</td>
</tr>
<tr>
<td>LB'(20)</td>
<td>28.01 (0.18)</td>
<td>15.95 (0.653)</td>
<td>25.22 (0.153)</td>
<td>25.81 (0.135)</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>3.09 (0.014)</td>
<td>1.21 (0.03)</td>
<td>1.80 (0.012)</td>
<td>0.41 (0.07)</td>
</tr>
<tr>
<td>JB</td>
<td>694.49 (0.000)</td>
<td>469.94 (0.000)</td>
<td>107.57 (0.000)</td>
<td>277.26 (0.000)</td>
</tr>
</tbody>
</table>

The autoregressive parameter of the conditional variance, $\beta$, is significant for all daily series, which lets think that FIGARCH modelling seems to be the most suitable one to describe the volatility of the daily return of our four assets.

The fractional parameter of integration $d$ is significant, which translates the fact that volatilities of the daily series are of long memory. FIGARCH modelling succeeded in taking into account the temporal dependence in the conditional variance because the statistics of Ljung Box of the squared residuals are nonsignificant.

All the parameters $d$, $\omega$, $\alpha$ and $\beta$, are statistically significant with the threshold of 5%.

All parameters $d$ in FIGARCH range between 0 and 0.5, which translates a persistence of volatility.

The choice of the distribution of Student is confirmed by the high value of $D$, since this degree of freedom is significant for all the studied series.

One of the stylized facts that marked the volatility of several financial series is the presence of a component of long memory in the conditional variance. This property, if it is correctly modelled, should make it possible to improve the forecast of future volatility.
In this case, it is now advisable to anticipate the trajectories in terms of volatility of the studied series by using FIGARCH modelling and this, at a 20 days horizon taking into account the dimension of short term of these markets.

It arises from the chart that since the mid-2008 peak, volatility (measured in terms of conditional variance) recorded successive falls (that appeared by a fall of the oil price from 147 dollars per barrel in July to 30 dollars at the end of December 2008).

Our forecasting represented in dotted line underlines an upturn of volatility, but at a less pace than in the past, and which appeared simultaneously in parallel with an increase of the prices (the price of oil displayed on November 6th, 2009 : 77.4 dollars/barrel, on December 16th : 80.2 dollars/barrel and on December 1st : 78.4 dollars/barrel).
The conditional variance of the S&P 500 Index informs us of a stabilization of the volatility over the period November 6th - December 1st, 2009 (either 20 days, except weekend).

Our anticipation is *a posteriori* validated on our horizon of forecast because a stabilization around 1100 points is really observed on this market at this period.

Our forecast announced stabilization around 1.48 dollars per euro over the period November 6th - December 1st, 2009. This foresee ability is also confirmed because the parity was contained in a range going from 1.48 to 1.50.

This conditional variance also indicated a stabilization of the volatility of the 10-Year U.S. interest rate around 3.40% on our forecast horizon. This prediction was also
carried out in the facts because this rate posted respectively: 3.54% on November 6th and 3.30% on December 1st, 2009.

**CONCLUSIONS**

The aim of this paper was to show which can be the interdependences, the causal links and the forecasting in terms of volatility on the following markets: Oil (WTI price), Foreign Exchange (nominal Euro/Dollar), Stock Market (S&P 500 Index) and Bond market (U.S.10-Year).

According to the results of VAR modelling, it arises that the four studied markets are interdependent in terms of volatility. From where a simultaneous study which requires to analyze the dynamics of the variations of these four markets, so to understand their volatilities.

The modelling of many statistical properties (normality, leptokurtic distribution, presence of phenomenon of heteroscedasticity, dependence of long run within volatility) of these series requires the use of a suitable model which can consider these characteristics in order to seize their dynamics as well as to minimize the risks of forecast errors. In fact, by considering these properties, the use of the FIGARCH II model seems to be the most relevant one in terms of forecasting. So, by applying this FIGARCH model we carried out 20 days horizon forecasts. It appears that our four anticipated conditional variances were validated *a posteriori* over the period from November 6th to December 1st, 2009.

**REFERENCES**
